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TECHNICAL NOTES

Transient conjugated heat transfer in laminar pipe flows

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INTRODUCTION

WITH the rapid advance of high technologies, the transient behaviour of an energy-related system during the period of startup, shutdown or any off-normal surge in a presumed steady, normal operation, possibly resulting from changes in loading conditions, becomes more and more important. Consequently, the study of transient heat transfer characteristics for fluid flowing in the various components of an energy system due to time variations in thermal conditions within the system has recently received considerable attention.

Because of the lack of efficient computational tools, the early attempts to treat unsteady heat transfer problems mainly employed approximate methods to deduce the gross features of the problems [1–3]. Recently, with the availability of large computing systems, numerical solutions have been obtained for unsteady thermal entrance heat transfer in laminar channel flows with different thermal boundary conditions [4–6].

The studies [1–6] reviewed above all focus on the transient heat transfer characteristics, neglecting unsteady wall heat capacity and wall conduction effects. The results thus produced are only good for heat transfer in flows bounded by extremely thin walls. However, in practical situations, the channel wall is finite in thickness, and the wall conduction and heat capacity effects should therefore be considered in the analysis. Recently, Sucec and Sawant [7–9] and Cotta *et al.* [10] examined the effects of wall heat capacity on unsteady heat transfer in laminar channel flows and showed that the duct wall heat capacity can have an important influence on the unsteady thermal characteristics in laminar channel flows; nevertheless, wall conduction still remains untreated. Recognizing the relatively little research on unsteady conjugated heat transfer, Chung and Kassemi [11], Krishnan [12] and the present authors [13] performed a study of transient conjugated heat transfer problems for flow over a flat plate or in a circular pipe. By an integral method, Chung and Kassemi [11] obtained the results for the unsteady conjugated heat transfer for flows over a flat plate. In ref. [12], the conjugated fully developed pipe flow with viscous dissipation effects was examined by the method of Laplace transforms, but its solutions are only valid for small values of time.

Despite the fact that unsteady conjugated heat transfer in laminar channel flows with stepwise variations in wall temperature is relatively important in engineering applications, it has not received much attention. This study aims to investigate the role of thermal resistance in connection with conduction heat transfer in the channel wall and unsteady heat capacity storage in the channel wall.

The geometry to be examined is a long circular pipe ($-\infty < x < \infty$) with inside radius R_i and outside radius R_o. The upstream portion of the pipe ($x < 0$) is insulated and long enough so that a Poiseuille velocity profile is obtained at $x = 0$. Initially, fluid and ambient are at the same uniform temperature T_e, and the flow enters the pipe at T_e. At $t = 0$, the temperature of outer pipe wall surface is suddenly raised to a new level T_{wo} and maintained at this level thereafter; heat exchange between the flow and the outer pipe wall then starts to occur.

ANALYSIS

As a preliminary study, axial conduction in both the fluid and the pipe wall is neglected. The conditions for the validity of this restriction are discussed in ref. [14]. By considering fluids with temperature-independent properties with laminar flow in the pipe and neglecting the viscous dissipation in the fluids, the energy transport processes in this problem are governed by the following non-dimensional equations:

energy equation for the fluid

$$\frac{\partial \theta_f}{\partial \tau} + (1-\eta^2)\frac{\partial \theta_f}{\partial \xi} = \frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \theta_f}{\partial \eta}\right); \quad (1)$$

energy equation for the pipe wall

$$\frac{\partial \theta_w}{\partial \tau} = \alpha_r \frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \theta_w}{\partial \eta}\right). \quad (2)$$

Equations (1) and (2) are subjected to the following initial and boundary conditions:

$$\tau = 0, \qquad \theta_f = \theta_w = 1 \quad (3a)$$

$$\xi = 0, \qquad \theta_f = 1 \quad (3b)$$

$$\eta = 0, \quad \partial\theta_f/\partial\eta = 0 \quad (3c)$$

$$\eta = \beta, \qquad \theta_w = 0. \quad (3d)$$

The matching conditions at the wall–fluid interface are described by

$$\eta = 1, \quad \theta_f = \theta_w \quad (4a)$$

$$\frac{\partial \theta_f}{\partial \eta} = K_r \frac{\partial \theta_w}{\partial \eta} \quad (4b)$$

where the dimensionless quantities are defined as

$$\xi = x/(R_i P_e), \quad \eta = r/R_i$$
$$\tau = \alpha_f t/R_i^2, \quad \theta = (T - T_{wo})/(T_e - T_{wo})$$
$$\alpha_r = \alpha_w/\alpha_f, \quad \beta = R_o/R_i$$
$$K_r = k_w/k_f. \quad (5)$$

In view of the impossibility of obtaining an analytic solution, as indicated in the literature survey, the problem defined by the foregoing equations was solved by finite-difference procedures. In the present study the matching condition imposed at the fluid–wall interface, equation (4b), so as to ensure the continuity of heat flux, was recast in backward difference for $\partial\theta_f/\partial\eta$ and forward difference for $\partial\theta_w/\partial\eta$. Therefore, the solution to the energy equations both in the fluid

NOMENCLATURE

K_r wall-to-fluid thermal conductivity ratio, equation (5)
k thermal conductivity
Q''_{wi} dimensionless interfacial heat flux
R_i inside radius
R_o outside radius
r radial coordinate
T temperature
T_{wo} outer wall temperature
t time
u_{ave} average fluid axial velocity
x axial coordinate.

Greek symbols
α thermal diffusivity
α_r wall-to-fluid thermal diffusivity ratio, equation (5)
β ratio of outside and inside radii, equation (5)
η dimensionless radial coordinate, equation (5)
θ dimensionless temperature, equation (5)
ξ dimensionless axial coordinate, equation (5)
τ dimensionless time, equation (5).

Subscripts
b bulk quantity
e initial value at the entrance of the pipe
f fluid
w pipe wall
wi fluid–wall interface.

and pipe wall can be solved simultaneously. A fully-implicit numerical scheme in which the unsteady energy storage term is approximated by the backward difference, axial convection by upwind difference and the radial diffusional term by the central difference is employed to transform the governing equations into finite-difference equations. This system of equations forms a tridiagonal matrix equation which can be solved efficiently by the Thomas algorithm [15]. It is noticed that drastic temperature variations are only present in the regions close to the thermal entrance and the fluid–wall interface for the initial, short period of time. Therefore, non-uniform grids are placed in both axial and radial directions along with the non-uniform time step. A total of 61×71 gridlines is placed in the x- and r-directions, respectively, and 101 time steps are used for the whole time duration. Computations with a doubling of the number of grid lines show that the results change by less than 2%.

RESULTS AND DISCUSSION

To verify the adequacy of the numerical scheme for the problem considered, the results for the temperature distributions for the flow in the region far from the entrance during the period of time within which the axial convection energy transport from upstream has negligible influence are obtained by the numerical scheme and by the analytic, closed form solution [16]. The solution corresponds to the so-called quasi-steady solution at the first time domain (i.e. $\tau < \xi$). Figure 1 shows the comparison between the results from these two different calculations. The agreement is good. Besides, it was found in separate numerical computations that agreements are equally good over wide ranges of the non-dimensional parameters β, K_r and α_r. Furthermore, the transient thermal entrance heat transfer in laminar pipe flows without wall conduction effects is found to be in excellent agreement with that obtained by Chen *et al.* [4]. These comparisons lend strong support to the employment of the scheme proposed to the study of the present problem.

The transient variations of the non-dimensional bulk fluid temperature θ_{fb} and fluid–wall interfacial temperature θ_{wi} are depicted in Fig. 2 for $\beta = 1.2$, $K_r = 10$ and $\alpha_r = 1$ at various axial locations. Both θ_{fb} and θ_{wi} decrease monotonically with τ and ξ—a typical pattern normally found for heat transfer in the entrance region. The results approach asymptotically the steady-state distributions with the pronounced variations for small τ. Suppose we focus our attention on the time variations of θ_{wi} at some axial location ξ, say $\xi = 0.1$. At very small τ, heat transfer is only by pure conduction, with

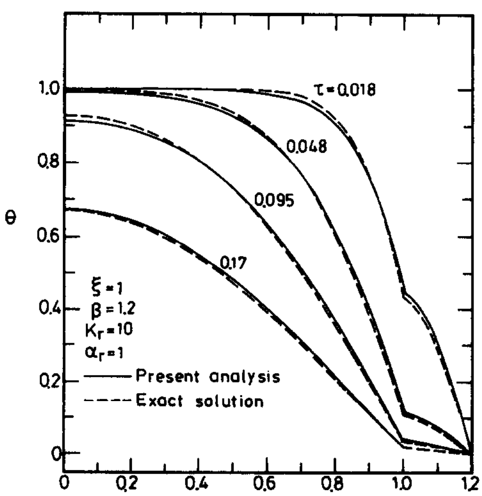

FIG. 1. Calculation of temperature profiles by numerical and analytic solutions.

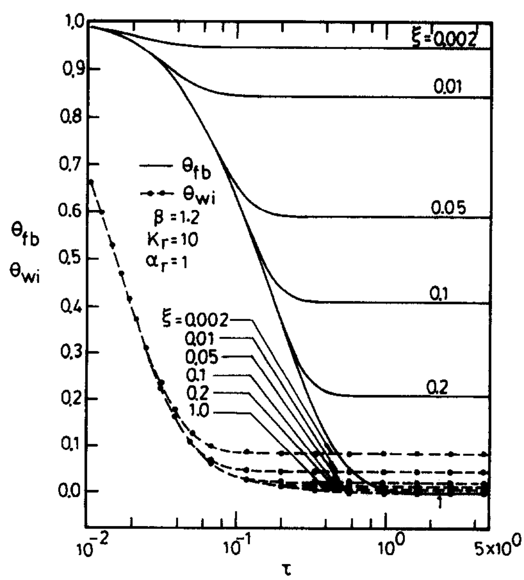

FIG. 2. The variations of the non-dimensional bulk fluid temperature and fluid–wall interface temperature.

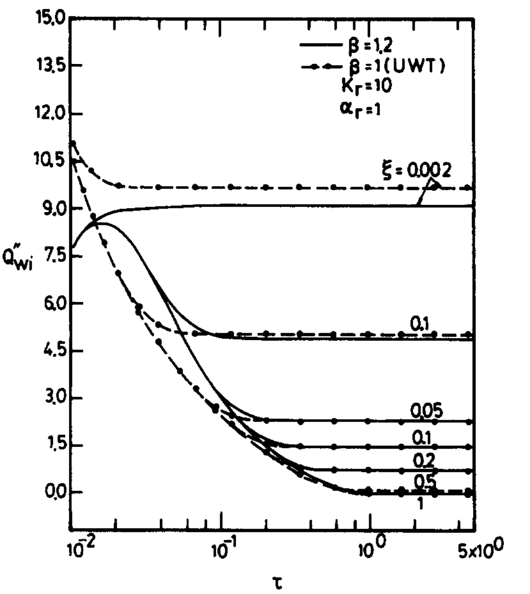

FIG. 3. The variations of the duct wall heat flux at the inside pipe surface.

the solution given in ref. [16], and θ_{wi} follows along the envelope curve, decreasing with increasing time. Then at $\tau = 0.1$ convection begins to act and the curve breaks away from the pure conduction envelope, with the interfacial temperature θ_{wi} continuing to decrease until the steady state is reached.

It is also noteworthy in Fig. 2 that as τ is small, θ_{wi} is far from the zero horizontal line which corresponds to the interfacial temperature distributions with no wall conduction, i.e. $\beta = 1$. This indicates that the effects of pipe wall conduction on the unsteady heat transfer characteristics are rather substantial, especially in the initial transient.

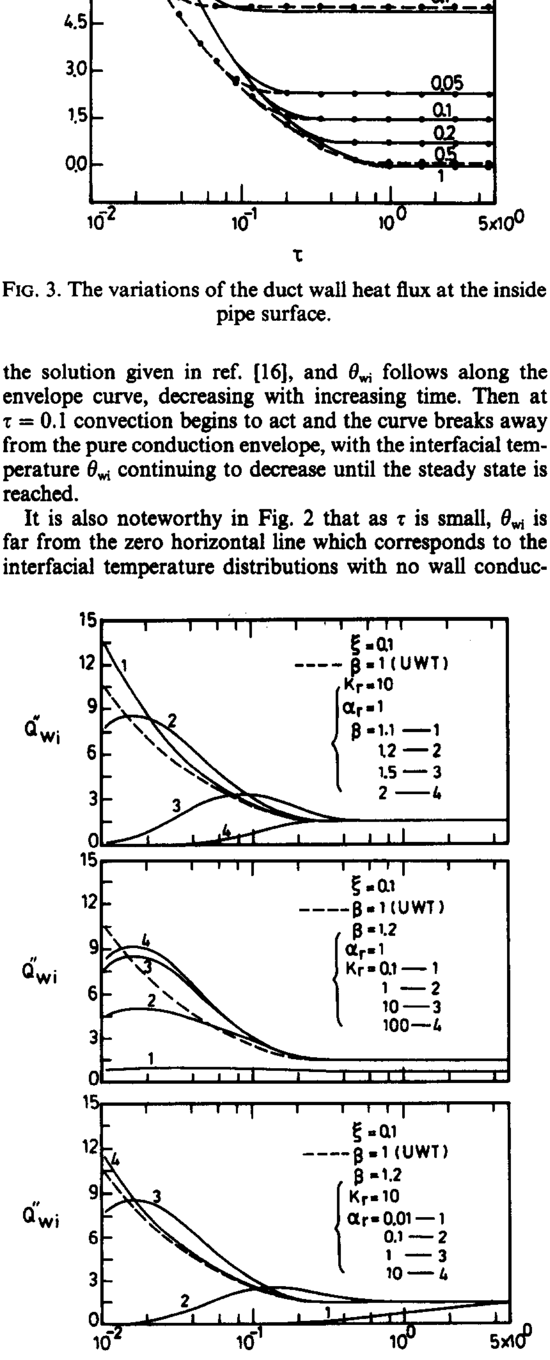

FIG. 4. Effects of α_r, K_r and β on the interfacial wall heat flux.

Figure 3 gives the time variations of the duct wall heat flux at the inside pipe surface, Q''_{wi}, which is defined as $2q''_{wi}R_i/[k(T_e - T_{wo})]$. Comparing the solid lines ($\beta = 1$) indicates that the deviation of the duct wall heat flux between them in the initial transient and in the entrance region is large. Further downstream and at large τ, the difference becomes very small. One point worth mentioning is that in Fig. 3 the time required for the heat transfer in the flow to reach the steady-state condition is a bit longer for the case with $\beta = 1.2$.

The influences of α_r, K_r and β on the distributions of the duct wall heat flux Q''_{wi} are illustrated in Fig. 4. It is clearly seen that parameters α_r, K_r and β have pronounced effects on unsteady heat transfer in laminar pipe flows, especially for the systems with larger β and smaller K_r and α_r. Therefore, it is unreasonable to assume that the wall may be thermally thin, so it is necessary to consider the wall effect in many analyses.

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